Math 104 **ACTIVITY 10:** Linear Programming: Formulating Problems and Graphing Feasible Regions

Why

The first step in solving a linear programming problem is the identification and formulation of the problem in standard form. Once the problem is in standard form, the next step is to see the *feasible region*—the set of points whose coordinates satisfy all the constraints. We will shortly see how to find the optimal solution (for a two-variable problem), based on the setup and the graph of the feasible region.

LEARNING OBJECTIVES

1. Be able to work effectively as a team, using the team roles
2. Be able to identify the decision variables in a linear programming problem.
3. Be able to interpret the constraints as algebraic inequalities (using the decision variables). identified)
4. Be able to identify and write the objective in algebraic form (using the decision variables)
5. be able to graph the individual constraints and combine these graphs to identify the feasible region.
6. Be able to locate the extreme points (corners) of the feasible region and find their coordinates.

CRITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in identifying the variables and representing the constraints and objective in linear programming situations.

RESOURCES

1. The course syllabus
2. The team role desk markers (handed out in class for use during the semester)
3. Your text – Sections 7.1–7.3
4. 50 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3.
2. Work through the exercises given here - be sure everyone understands all results & procedures.
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade.

DISCUSSION

I. The setup:
A linear programming problem asks for a decision about two or more quantities. Their sizes are represented by the *decision variables* of the problem (the name emphasizes that these are the variables whose values represent the decision to be made). To determine the decision variables, you need to determine *what must be decided*; in a textbook situation, the question will often directly ask for these values.

The second step in formulating the model is identifying the *objective* and representing its value in terms of the decision variables. The question is, "What is the measure of success?" Usually this can be thought of as the benefit from whatever we are planning (to be maximized) or some sort of cost (to be minimized).

The *constraints* express the conditions that limit our choice of values for the decision variables. Often there are minimum requirements for results (as in the diet problem) or limits on resources that must be used (as in the toy car and truck production problem). In virtually every situation, we must add in the requirement that the variables cannot be negative

Once the variables are identified, it is often helpful to think of making a *table* with a column for each variable and a row for each constraint (each condition that must be fulfilled or supply that is limited) plus a row for the objective. At the right of each row, we can put the limiting amount (requirement or amount available or whatever limit) with the appropriate inequality ($\leq$ for amount available of a resource used, $\geq$ for an amount required to be provided, etc.). Then the coefficients (amount used or produced per unit of the variable) can be filled in, and the constraints and objective read from the table. If the information for the problem is given in a table, the *rows* of that table often correspond to the variables, and will become the *columns* of this setup table. There may also be more direct relations between the variables (That one must be at least twice as large as another, for example).

II. The graph
The feasible region is the region where the solution sets for all the constraints overlap (the intersection of all the solution sets of the constraint inequalities). To graph it:
For each constraint: Look at the *equation*—it gives the boundary for the solution set. We graph this by finding two points that satisfy the equation, and drawing the line through them. (Easiest - let $x = 0$, find corresponding y, then let $y = 0$, find corresponding x. This are usually the easiest points to graph , as well as the easiest to calculate).
After the line is drawn, determine which *side* is the solution region (Easiest way – pick a point not on the line—use $(0, 0)$ if possible, to simplify the arithmetic—and see if it works in the inequality. If so, we want that side of the line; if not, we want the other. Shade the appropriate side lightly.
For a linear programming problem, we will always have $x \geq 0$ and $y \geq 0$ among the constraints - so our region will always be in the first (positive) quadrant of the coordinate plane.
The *corners* come from intersections of (some of) the lines — we solve the pair of equations to find coordinates.

MODELS

1. Exercise 1 from p.203 (Section 7.1) in the text:
   SETUP
   Variables: We want to know/decide how many sandwiches of each type to make (types are **1.** "small' and **2.**"large"), so our *variables* are
   number of small sandwiches
   number of large sandwiches
   "More success" will mean "larger profit", so our *objective is to maximize profit* at \$0.80 per small sandwich and \$1.20 per large sandwich
   We have limitations/requirements based on the amount of bread and the amount of meat available and the amounts used in the sandwiches. We can organize the information by writing:

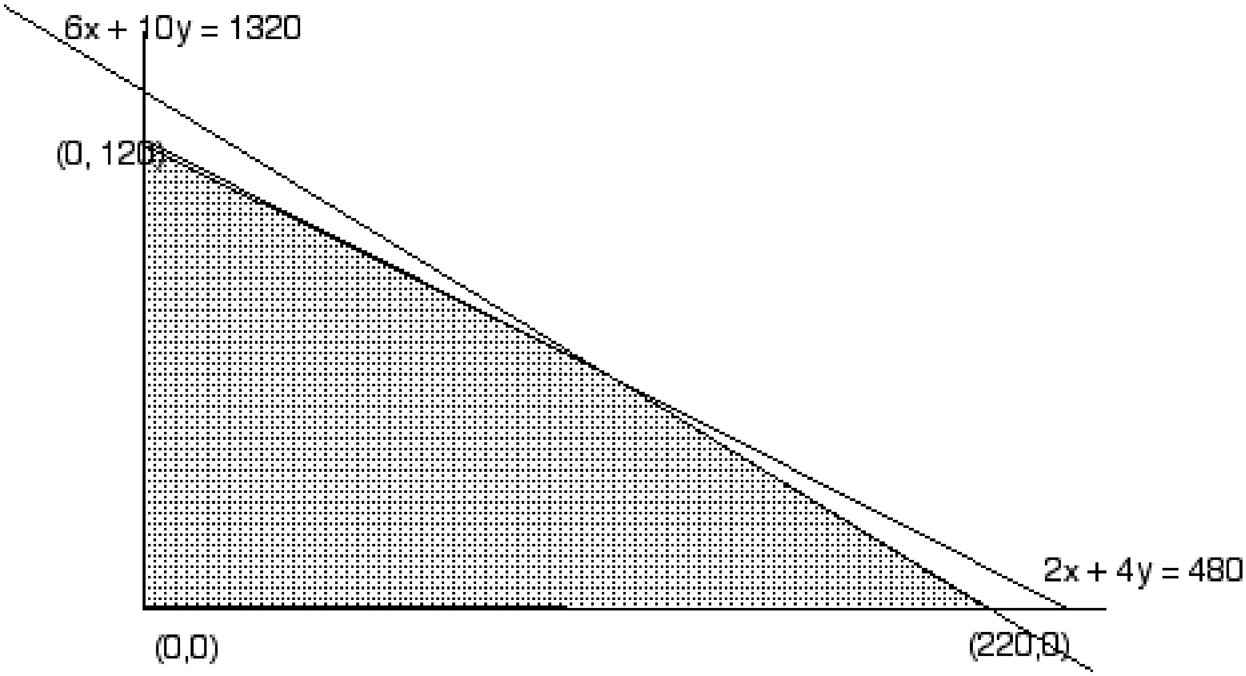
	small	large	
profit	\$0.80	\$1.20	maximize
bread	6 in.	10 in.	$\leq$ 110 ft. $\times$12 inches/ft. = 1320 in.
meat	2 oz.	4 oz.	$\leq$ 30 lb $\times$16 oz./lb. = 480 oz.

   so our setup is:

   Variables:
   x = number of small sandwiches
   y = number of large sandwiches
   Objective:
   profit = $0.80x + 1.20y$ maximize
   Subject to **Constraints:**

bread	$6x$	$+ 10y$	≤ 1320
meat	$2x$	$+ 4y$	≤ 480
	x		≥ 0
		y	≥ 0

   GRAPH
   The equation for the first constraint is $6x + 10y = 1320$. To get two points, we choose $x = 0$ and find, for the line, $y = 1320/10 = 132$ and we choose $y = 0$ and find the corresponding $x = 1320/6 = 220$ so this line goes through points $(220, 0)$ and $(0, 132)$. The point $(0, 0)$ is not on the line and $6(0) + 10(0) = 0 \leq 1320$, so we want the $(0, 0)$ side of this line.
   For $2x + 4y \leq 480$ we similarly get points $(0, 120)$ and $(240, 0)$ and find that we again want the $(0, 0)$ side of this line.
   The constraints $x \geq 0$ and $y \geq 0$ keep us in the first quadrant of the plane.
   Our graph is:



To obtain the coordinates of the fifth corner, we notice that it is formed at the crossing of the lines for the equations $6x + 10y = 1320$ and$2x + 4y = 480$, so we solve this pair of equations:
If we multiply the second equation by -3, we obtain $-6x - 12y = -1440$; adding this to the other gives $-2y = -120$, so at the intersection point we know $y = 60$. Substituting this into the second equation gives $2x + 4(60) = 480$, so $x = 120$ and the corner is the point $(120, 60)$
Thus our corners are $(0, 0), (220, 0), (120, 60)$ and $(0, 120)$.

2. Exercise 6 (includes information for exercise 5) on p. 204(section 7.1) in text:
Variables: The company must decide how many gallons of each type of syrup to make, so the variables will be:
number of gallons of regular syrup
number of gallons of Extra Maple syrup
"More success" will correspond to "larger profit", so out objective is to maximize profit, at \$3 for each gallon of regular syrup and \$5 for each gallon of Extra Maple syrup.
We have limitations based on the amount of maple base available, the amount of sugar available, and the amount of Extra Maple that can be sold. There is also (form #6) a limitation based on a direct relationship between the variables. We can organize the information from the objective and the first three conditions in a table; the fourth condition is handled a little differently:

	regular	Extra	
profit	\$3	\$5	maximize
base	5	2	$\leq 10,000$ gallons
sugar	2	4	≤ 8800 pounds
market	1	0	≤ 1800 gallons

amount regular $\leq$ amount Extra—which we can write as $x \leq y$ and manipulate to $x - y \leq 0$
so our model is:

Variables:
$x =$ number of gallons of regular syrup
$y =$ number of gallons of Extra Maple syrup
Objective:

profit =	$3x$	$+ 5y$	maximize

subject to **Constraints**:

base	$5x$	$+ 2y$	$\leq$10,000
sugar	$2x$	$+ 4y$	$\leq$8800
market	x		$\leq$1800
production	x	$- y$	$\leq$0
	x		$\geq$0
		y	$\geq$0

EXERCISES

1. Set up (give variables, objective, constraints) exercise #22 on p.208.
    (a) Give two possible combinations of science fiction and fantasy scenarios and give the profit for each.
    (b) Give two combinations of science fiction and fantasy scenarios that cannot be used, and give the (theoretical, if they could be used) profit for each.
    (c) Graph the feasible region and find the coordinates of all the corners (Some pairs of lines do not give corners—we don't care about intersections that don't give corners)
2. Set up (give variables, objective, constraints) exercise #12 on p.205.
    (a) If the company can run 5 trips on the North Fork and 25 trips through Blue River Gorge, what profit will they gain? Is this combination of trips possible?
    (b) If the company can run 10 trips on the North Fork and 10 trips through Blue River Gorge, what profit will they gain? Is this combination of trips possible?
    (c) Which of these is a better choice for the company? Why?
    (d) Graph the feasible region and find the coordinates of the corners.

READING ASSIGNMENT (in preparation for next class meeting)

Read section 7.3 on solving two-variable linear programming problems

SKILL EXERCISES:(hand in - individually - at next class meeting)

p.203 #13 – 15, p.215 # 24, 31–32