Why

Continuous random variables require different methods for calculation and description of their distributions. The normal distributions provide the basis for most [but not all] of the inferential statistics you have studied, and are important for many models. The exponential family, related to waiting times, provides another example and is important for its relation to the poisson distribution and to waiting times.

LEARNING OBJECTIVES

1. Work as a team, using the team roles
2. Recall fundamental ideas related to continuous probability distributions.
3. Be able to identify situations which can be modeled by a normally distributed variable and carry out probability calculations there.
4. Be able to identify situations which can be modeled by an exponentially distributed variable and carry out probability calculations there.

CITERIA

1. Success in completing the exercises.
2. Success in working as a team

RESOURCES

1. Your Text - especially chapter 6
2. Probability tables in your text: standard normal pp. 918-919
3. The discussion below
4. Your Calculator
5. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the exercises given here - be sure everyone understands all results (30 minutes)
3. Assess the team’s work and roles performances and prepare the Reflector’s and Recorder’s reports including team grade (5 minutes).
4. Be prepared to discuss your results.

DISCUSSION

For a continuous variable, we cannot list the possible values or assign probabilities to individual values. Instead we use a probability density function - a never-negative function whose graph has a total area (beneath the graph) of 1. We can only discuss probabilities for intervals - the probability of getting a value in a particular interval is the area above that interval and under the graph of the density function.

The normal distribution models many variables that are the sum of many independent causes [thus the connection to both the distribution of sample means and the large-\(n\) binomial]. A particular normal distribution is specified by its mean \(\mu\) and standard deviation \(\sigma\). For calculations we use a table [see pp. 918-919] giving the cumulative distribution function \(P(Z < z)\) for many values of the standard normal distribution (with \(\mu = 0\) and \(\sigma = 1\)); use of the table relies on the fact that for any normally distributed variable \(X\), the standardized variable \(Z = \frac{X-\mu}{\sigma}\) fits the standard normal distribution.
The normal distribution also is used to approximate the distribution of a binomial variable with a large enough number of trials \( n \); in fact the normal distribution formulas were originally developed as approximations to the large binomial. The usual rule of thumb is that \( n \) must be large enough (and \( p \) close enough to .5) that both \( np \) and \( n - np \) are 5 or more [approximation is pretty rough at this extreme]. Since the binomial is discrete (probability assigned to each possible value) and the normal is continuous (probability assigned only for intervals) we also reinterpret the binomial condition “\( X = k \)” as normal condition “\( k - .5 < X < k + .5 \)” (the continuity correction).

The exponential distribution is used to model many waiting time situations – especially those in which the occurrence of events follows a poisson distribution. It also applies to “time to failure” or lifetime calculations for many manufactured items. The probability distribution function is given by \( f(x) = \frac{1}{\mu} e^{-x/\mu} \) (\( \mu \) is the mean) and we can directly calculate the distribution function using \( P(X < x) = 1 - e^{-x/\mu} \) (no table needed). There is a close relation between the poisson and exponential distributions: If \( X \) gives the “number of occurrences in a unit of time” (number of customers coming to a bank window in one hour, for example) and follows a poisson distribution with mean \( \lambda \) (customers per hour), then the “time until the next occurrence” (time between customers) follows an exponential distribution with mean \( \mu = 1/\lambda \) (hours between customers).

**EXERCISE**

1. A manufacturing process produces items whose length is approximated by a normal distribution with a mean length 10 cm and a standard deviation of lengths .15.
   
   (a) What proportion of items produced are more than 10.2 cm long?
   
   (b) What range of lengths covers the middle 90% of the items produced?
   
   (c) The process control is set at .15 cm - that is, any item whose length is not between 9.85 cm and 10.15 cm is classed as defective and must be discarded. What proportion of items are defective?
   
   (d) If the process can be refined so that the standard deviation of lengths is reduced to .10 cm, what proportion of items will be defective (with the same process control)?

2. At a time when the unemployment rate is at 6.2%, a survey in a large city involves interviews with 100 people.
   
   (a) Can we use the normal approximation to the binomial to discuss probabilities for the number of unemployed people who are interviewed? Why/why not?
   
   (b) What is the probability that 10 or more unemployed people will be interviewed?

3. The mean time to clear the security checkpoint at peak periods in Smalltown airport is 12.2 minutes, approximately exponentially distributed.
   
   (a) What is the probability it will take more than 20 minutes to clear the security checkpoint at a peak period?
   
   (b) What is the probability it will take between 10 and 20 minutes to clear the security checkpoint at a peak period.
   
   (c) If you arrive at the checkpoint a half hour before you must board your flight and it takes 12 minutes to get from the checkpoint to the gate, what is the probability you will miss the flight?

4. At first National Bank, customers the arrival of customers at the drive-up window fits a poisson distribution with mean eight customers per hour.
   
   (a) What is the probability that six or fewer customers will arrive in a particular one-hour period?
   
   (b) What is the average length of time between arrivals?
   
   (c) When the supervisor comes to the window to observe a transaction, what is the probability she will have to wait fifteen minutes for the next customer?

**READING ASSIGNMENT** (in preparation for next class)
Read Sections 20.1-20.2 on statistical process control – we will look at this application of normal probability models on Friday.

**SKILL EXERCISES:** (hand in - individually - at next class meeting): p.251: #46, 49 – 53