ACTIVITY 6: Inference on two population variances – the F distribution

Why

We continue extending our scope for inference - now we look at the two-population case for inference on variances. We will look only at tests (Do we have evidence of a difference in variances/standard deviations?) and will not consider confidence intervals, which would be very hard to interpret. This topic also introduces a new important distribution – the F distribution, which arises from the ratio of two $\chi^2$ variables.

LEARNING OBJECTIVES

1. Work as a team, using the team roles
2. Understand the test situation for comparing variances/standard deviations
3. Understand the F-distribution and be able to determine critical values for our tests.
4. Understand the significance test model in another situation.

CRITERIA

1. Success in completing the exercises.
2. Success in answering questions about the model
3. Success in working as a team

RESOURCES

1. The document “Inference (hypothesis tests and estimation): Variance of one population, comparing two variances” handed out Monday
2. The document “Hypothesis testing – generalities”
3. Your Text - especially section 11.2
4. The model below
5. Your calculator
6. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the exercises given here - be sure everyone understands all results (30 minutes)
3. Assess the team’s work and roles performances and prepare the Reflector’s and Recorder’s reports including team grade (5 minutes).
4. Be prepared to discuss your results.

MODELS

1. Reading the F-table (p.925)

(a) To carry out a test with alternate hypothesis $\sigma_1^2 > \sigma_2^2$, with $\alpha = .05$, if the sample size for population 1 is 31 and for population 2 is 24, we need $F_{.05, 30, 23}$ (the last numbers representing the degrees of freedom for numerator and denominator, respectively). So our test criterion will be “Reject $H_0$ if sample $F > 1.96$”. If we obtain an F-value 2.50, then our p-value will be between .025 and .01 (because 2.50 is between 2.24 – the critical value for $\alpha = .025$ – and 2.62 – the critical value for $\alpha = .01$).
(b) To carry out a test with alternate hypothesis $\sigma_1^2 < \sigma_2^2$, with $\alpha = .05$, if the sample size for population 1 is 21 and for population 2 is 16, we need $F_{.95 \frac{20}{25}}$. The table does not include left-side values such as $F_{.95 \frac{15}{15}}$, so we need to use the conversion formula on the handout: We use the reciprocal and the complementary probability and swap the degrees of freedom: $F_{.95 \frac{20}{25}} = \frac{1}{F_{.05 \frac{25}{20}}} = \frac{1}{.20} = .50$. So our test criterion will be “Reject $H_0$ if sample $F < .50$”. [Left-side critical values for $F$ will typically be less than 1]

2. We are comparing two types of thermostats — we want to know if either one provides a more even temperature than the other. We run a series of tests in which the thermostats are used to control the temperature of a room, which is supposed to be kept at 72 degrees Fahrenheit, and we take readings of the actual temperature at regular intervals. We will test for a difference in variability. For the “super climate” thermostat, we obtain 41 readings with mean 73.95 degrees and standard deviation 3.82 degrees. For the “heat master” thermostat, we obtain 26 readings with mean 71.96 degrees and standard deviation 2.56 degrees. For the “heat master” we obtain 26 readings with mean 71.96 degrees and standard deviation 3.82 degrees. Does this give evidence at the .05 level that there is a difference between the two thermostats in the variability of the temperatures?

I Variable: $X_A =$ room temperature under control of “super climate”, $X_B =$ room temperature under control of “heat master”. Test is:

- $H_0 : \sigma_A = \sigma_B$
- $H_a : \sigma_A \neq \sigma_B$

II Statistic $F = \frac{s_A^2}{s_B^2}$ with $df_{\text{numerator}} = 40, df_{\text{denominator}} = 25$

III To test at .05 level: Reject $H_0$ if sample $F < F_{.95 \frac{40}{25}}$ or if sample $F > F_{.05 \frac{40}{25}}$. Using table $F_{.05 \frac{40}{25}} = 2.12$, and from formula $F_{.95 \frac{40}{25}} = \frac{1}{F_{.05 \frac{25}{40}}} = \frac{1}{1.99} = .503$. That is, we will reject $H_0$ if $F < .503$ or if $F > 2.12$.

IV sample $F = \frac{2.562}{3.82} = .449$ which is less than $F < F_{.95 \frac{40}{25}} = .503$

V We reject $H_0$

VI The sample shows, at the .05 level, that there is a difference in variance of temperature between the two thermostats.

EXERCISE

1. Using the F-table

(a) If we are testing for the alternative hypothesis $\sigma_1^2 > \sigma_2^2$ at the $\alpha = .05$ level, with $n_1 = 25$ and $n_2 = 30$, what are our test value and criterion (“Reject $H_0$ if …”)?

(b) If we are testing for the alternative hypothesis $\sigma_1^2 < \sigma_2^2$ at the .025 level, with $n_1 = 25, n_2 = 15$, what are our test value and criterion?

(c) If we are testing for the alternative hypothesis $\sigma_1^2 \neq \sigma_2^2$ at the .025 level, with $n_1 = 16, n_2 = 25$, what are our test values and criteria?

2. We want to know if investment in stock A carries less risk (lower standard deviation in price) than stock B. We have data for 31 days of prices of stock A, giving mean $38.67 and standard deviation $3.79$ and for 26 days of prices of stock B, giving mean $42.75$ and standard deviation $5.55$. Does this data give evidence that the standard deviation of prices for A is less than the standard deviation for B?

3. A pet food canning factory operates several production lines; operations managers suspect that the variability of the weights of the cans is greater on production line A. Random samples of cans from the two lines give the following information:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line A</td>
<td>8.005</td>
<td>0.012</td>
<td>11</td>
</tr>
<tr>
<td>Line B</td>
<td>7.997</td>
<td>0.005</td>
<td>16</td>
</tr>
</tbody>
</table>

Do the data give evidence of a difference more variability on line A? What is the p-value?

READING ASSIGNMENT (in preparation for next class)
Read Chapter 12 sections 12.1 and 12.3 — inference (tests) for a single distribution ($\chi^2$ goodness of fit)

SKILL EXERCISES: Use your calculator or Minitab for number -crunching [Minitab will carry out hypothesis tests when you have actual data to work with] but you have to write the hypotheses and conclusion. p.451 #18–19, p.451 #25, 31