

Why

These are three important theorems involving the division of chords, secants and tangents. In essence, they are three cases of the same relation. All deal with the lengths of segments determined by the intersection of two lines with each other and with a circle. The cases depend on whether the intersection of the lines is in the interior or the exterior of the circle and, in the second case, whether one line is tangent to the circle or both are secants. The corollary to the tangent theorem (3.8.2) is a very special case for the same relation with both the lines tangent to the circle so there are only two segments (rather than three or four).

LEARNING OBJECTIVES

1. Continue to develop skill in working as a team
2. Develop further skill in using a computer geometry system.
3. Gain experience in making a conjecture from experience with some geometric examples.
4. Gain skill in writing formal proofs

CRITERIA

1. Success in working as a team in generating ideas, correcting errors, and developing understanding
2. Success in completing the exercise
3. Accuracy and completeness (and explanation) of your proofs

RESOURCES

1. Your text, especially sections 4.3 and 4.5
2. The outline notes
3. 40 minutes
4. The Geometer's Sketchpad program -on the campus network

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the exercises given here - be sure everyone understands all results (30 minutes)
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).
4. Be prepared to discuss your results.

EXERCISE

1. Carry out the Sketchpad construction given here, and manipulate the objects as indicated, writing down your observations.
2. For each of these three cases, write a conjecture, (a general result regarding relationships on segment lengths)
 - I.) Two chords meeting in the interior of a circle
 - II.) Two secants meeting at a point exterior to the circle
 - III.) A secant and tangent meeting at a point exterior to the circle.

- 3.) Write out a proof of one of your conjectures, using the results that we studied in section 4.3 and the inscribed angle theorem in section 4.5. [A hint for case I or II – consider triangles $\triangle EPH$ and $\triangle GPF$ (names as shown in construction below) You can calculate angle and arc measures with Sketchpad to check parts of your conjecture – but that won't constitute *proof*, just as the calculations requested here don't make up a proof]. Remember that $\frac{a}{b} = \frac{c}{d}$ is algebraically equivalent to $\frac{a}{c} = \frac{b}{d}$ and to $ad = bc$, etc. – so if there is one relation that holds it should show up in several ways. Those won't require separate proofs. Could your proof be modified to cover the other two situations [if you write a proof for your conjecture in Case I, could it be modified to prove the result in Case II]?

The construction : [Not detailed instructions]

Construct a diagram that can be used to consider lengths of segments created by the intersection of two chords, by the intersection of two (intersecting) secants with a circle, and by a tangent and an intersecting secant.

- a.) Draw a circle with center A
- b.) Create a point P in the interior of the circle
- c.) Use the point tool to construct a point E on the circle (the circle is highlighted when the point will stick to it), and construct the line \overleftrightarrow{PE}
- d.) Use the point tool to construct a point F at the intersection of \overleftrightarrow{PE} and the circle [Using the “Construct Point at Intersection” command will construct *two* points and make it difficult to manipulate the sketch, so you don't want to do that] These two steps give a chord \overline{EF} on the line \overleftrightarrow{PE}
- e.) Repeat c.) and d.) to get a line \overleftrightarrow{GP} and chord \overline{GH} containing P .
- f.) Use the Measure>Distance command to find distances PE, PF, PG, PH
- g.) Use Measure>Calculator to find the products $PE \cdot PF, PG \cdot PH, PE \cdot PG, PF \cdot PE$ and the ratios $PE/PG, PF/PH, PE/PH, PF/PG, PG/PF, PG/PE, PH/PE, PH/PF$ and display them on the sketch.

The manipulations:

- a.) [Case I] Move points E and G to various points on the circle [Because the “second points” F and H are determined by the “first points” E and G , you can't drag them in any interesting way]. What do you observe? [What values change? What values do not change? Are any pairs of values equal in all cases?] [Of course $PE/PG = 1/(PG/PE)$, etc. – those aren't worth mentioning]
- b.) [More Case I] Move point P in the interior of the circle. What do you observe? [Same ideas]
- c.) [Case II] Move P to the exterior of the circle so that you have two secants intersecting at P . (Be careful, here – drag P out over F and H . Otherwise G and H (or E and F) may stick together and give incorrect measurements and ratios) Try different locations for P, E and G . What do you observe?
- d.) [Case III] Move P so that G and H coincide (giving a tangent) [You only get a tangent when they first coincide – if you keep moving P the measurements are inaccurate because G and H stick to each other]. There is also sometimes a roundoff problem in the calculated values]. Try different positions for E , for P (but with G & H coincident) What do you observe?

SKILL EXERCISES:(hand in - individually - with this week's assignments)

Write: text p 277 # 1, 2