Math 438
Sample Test 2
2010

Show your work—partial credit is available for work shown, and unsupported answers will not usually receive full credit.
Attach this question sheet to the back of your answer sheets.
Numbers in parentheses indicate point value of each problem or part.

1. (10) Answer two of these
   (a) In using the simplex method, how do we know when we have an unbounded problem?.
   (b) What are the possible combinations for feasibility, boundedness, solvability for a Primal and Dual pair of LP problems [Duality theorem]
   (c)) What is the form of a Symmetric Primal Linear Programming problem ?
   (d) How can we tell, from the final tableau in the simplex method, if there are alternate optimal solutions? Why does this test work?

2. (15) Consider the following LP problem in canonical form:

$$\begin{array}{lrl} \text{maximize} & -5x_1 + 3x_2 - x_3 & \\ \text{subject to} & 2x_1 + x_2 - x_3 - x_4 & = 5 \\ & x_1 - x_2 + x_3 & = 3 \\ & x_1 + x_2 \qquad\qquad + x_5 & = 6 \\ & \text{all } x_i \geq 0 & \end{array}$$

   (a) How can you tell that we need artificial variables (the two-phase simplex method) to solve this problem? How many are needed?
   (b) Here is the initial tableau for phase I. Determine whether the problem is feasible.
   If the problem is feasible, write the initial tableau for phase II (do not carry out the solution)
   If the problem is not feasible, say how you can tell.

$$\begin{bmatrix} 2 & 1 & -1 & -1 & 0 & 1 & 0 & 5 \\ 1 & -1 & 1 & 0 & 0 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

3. (15) For this LP problem:

$$\begin{array}{lrl} \text{miniimize} & -3x_1 - 2x_2 + 2x_3 & \\ \text{subject to} & x_1 + x_2 \qquad & \leq 5 \\ & x_1 - x_2 + x_3 & \leq 8 \\ & x_2 + x_3 & \leq 4 \\ & \text{all } x_i \geq 0 & \end{array}$$

   (a) Write the problem in canonical form
   (b) Determine which variables form the initial basis and give their (initial) values:
   (c) Determine which non-basic variable should enter the basis and which basic variable will leave. (Show how you decide).

4. (15) On page 3 you will find the initial and final tableaus and the sensitivity analysis for problem 3.
   (a) Give the optimal solution and the value of the (original) objective function.
   (b) How far could we increase the objective function coefficient on x_1 (it is now 3) without changing the optimal solution? [The value would change, of course] How can you tell?
   (c) If the objective function coefficient on x_3 increased from -2 to 5, would this affect our optimal solution or the value? How can you tell?

5. (10) Working with the problem in #3 (and the tableaus shown on p.3):
   (a) Write the dual problem in standard form

(b) Give the solution and value (of the objective) for the dual from the information given. [No major calculation is required]

6. An electronics firm produces three types of switching devices. Each type involves a two-step assembly operation. The assembly times are shown in the following table:

	Assembly time per unit (minutes)	
	Station #1	Station #2
Model A	2.5	3.0
Model B	1.8	1.6
Model C	2.0	2.2

The working time available at each work station is only 7 hours (not minutes). Model A yields a profit of \$8.25 per unit, Model B a profit of \$7.50 per unit, and Model C a profit of \$7.80 per unit. The company can sell at most 150 units of each type of switching device. They wish to plan production to maximize daily profit.

(a) (10) Write this problem as an LP problem in canonical form, using x_1 = number of units of A, x_2 = number of units of B , x_3 = number of units of C, and numbering the slack variables in the order in which the restrictions are given(Station 1 time, Station 2 time, limit on model A , etc.). Set up the simplex tableau.

(b) (5) Given the final tableau shown here, give the optimal production plan and optimal profit

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 150 \\ 125 & 0 & 1 & .5 & 0 & 0 & -.9 & 0 & 75 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 150 \\ .25 & 0 & 0 & -1.1 & 1 & 0 & .38 & 0 & 15 \\ -1.25 & 0 & 0 & -.5 & 0 & 0 & .9 & 1 & 75 \\ 15 & 0 & 0 & 3.9 & 0 & 0 & .5 & 0 & 1710 \end{bmatrix}$$

(c) (10) Give $\mathbf{B}, \mathbf{D}, \mathbf{C_B}, \mathbf{C_D}, \mathbf{B^{-1}D}, \mathbf{B^{-1}}$, and $\mathbf{C_B^T B^{-1} D} - \mathbf{C_D^T}$

(d)) (5) What would be the effect on the profit of producing 5 units of Model A? How can you tell?

(e) (5) What is the shadow price for time available at work station #1? What does it tell you ?

(f) (5) Suppose that bottlenecks in the shipping department limit production to a total of 300 switching devices per day. Will this additional constraint change either the optimal production plan or the optimal profit? How can you tell?

105 points total

Tableaus and sensitivity analysis for Question 3

Initial tableau:

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 5 \\ 1 & -1 & 1 & 0 & 1 & 0 & 8 \\ 0 & 1 & 1 & 0 & 0 & 1 & 4 \\ -3 & -2 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Final tableau:

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & -2 & 1 & -1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 2 & 3 & 0 & 0 & 15 \end{bmatrix}$$

Microsoft Excel 11.6 Sensitivity Report
Worksheet: [Workbook1]Sheet1
Report Created: 11/8/2010 11:59:52 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C4	x1	5	0	3	1E+30	1
D4	x2	0	-1	2	1	1E+30
E4	x3	0	-2	-2	2	1E+30
F4	s1	0	-3	0	3	1E+30
G4	s2	3	0	0	0.5	2
H4	s3	4	0	0	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I6		5	3	5	3	5
I7		8	0	8	1E+30	3
I8		4	0	4	1E+30	4