# Analysis Lab 9

## Topic: Continuity and Sequences

### Guidelines for Lab Report

For this lab, submit a report according to guidelines given below.

1. For Section 3, fill in each cell of the table provided on page 2 of this report guide.
2. For Section 4, submit your answers to Questions 1-7. The tables for Questions 5 and 6 are provided on page 4.
3. Complete the Questions for Reflection as assigned by your instructor. Write your response to each question on a separate sheet(s), and attach to the rest of this report.

## 3 Using Examples to Enhance Understanding

| $i$ | $f_i$, $x_0$, Sequences | Q1 | Q2 | Q3 | Q4 | Q5 |
|---|---|---|---|---|---|---|
| 1 | $f_1(x) = x^2 - 1, \quad x_0 = 0$ <br> $(a_n)_{n=1}^{\infty} = \left(\frac{1}{n}\right)_{n=1}^{\infty} \qquad (b_n)_{n=1}^{\infty} = \left(\frac{n}{n^2+1}\right)_{n=1}^{\infty}$ | | | | | |
| 2 | $f_2(x) = \begin{cases} x-2, & \text{if } x < 4 \\ 2, & \text{if } x = 4 \\ 6-2x, & \text{if } x > 4 \end{cases}, \quad x_0 = 4$ <br> $(a_n)_{n=1}^{\infty} = \left(4 + \frac{(-1)^n}{n}\right)_{n=1}^{\infty} \qquad (b_n)_{n=1}^{\infty} = \left(4 - \frac{1}{n}\right)_{n=1}^{\infty}$ | | | | | |
| 3 | $f_3(x) = \begin{cases} 1/x, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}, \quad x_0 = 0$ <br> $(a_n)_{n=1}^{\infty} = \left(-\frac{1}{n}\right)_{n=1}^{\infty} \qquad (b_n)_{n=1}^{\infty} = \left(\frac{1}{n^2}\right)_{n=1}^{\infty}$ | | | | | |
| 4 | $f_4(x) = \begin{cases} \frac{x^2-2x-15}{x-5}, & \text{if } x \neq 5 \\ 1, & \text{if } x = 5 \end{cases}, \quad x_0 = 5$ <br> $(a_n)_{n=1}^{\infty} = \left(\frac{5n}{n+1}\right)_{n=1}^{\infty} \qquad (b_n)_{n=1}^{\infty} = \left(5 + \frac{(-1)^n}{n}\right)_{n=1}^{\infty}$ | | | | | |
| 5 | $f_5(x) = \begin{cases} 7-x, & \text{if } x < 2 \\ 2x+1, & \text{if } x \geq 2 \end{cases}, \quad x_0 = 2$ <br> $(a_n)_{n=1}^{\infty} = \left(2 - \frac{1}{n}\right)_{n=1}^{\infty} \qquad (b_n)_{n=1}^{\infty} = \left(\frac{2n}{n+3}\right)_{n=1}^{\infty}$ | | | | | |

## 4 Critical Thinking Questions

In the space provided, write your answers to Questions 1-4.

5.

| $g_i$ | $g_i(x_0)$ | Behavior of $(g_i(x_n))_{n=1}^{\infty}$, where $(x_n)_{n=1}^{\infty}$ converges to $x_0$ | Continuous at $x_0$? |
|---|---|---|---|
| $g_1$ | 2 | There exists $(x_n) \longrightarrow x_0$ such that $\lim_{n\to\infty} g_1(x_n)$ DNE. | |
| $g_2$ | DNE | For all $(x_n) \longrightarrow x_0$, $\lim_{n\to\infty} g_2(x_n) = 3$. | |
| $g_3$ | 3 | | C |
| $g_4$ | -2 | | NC |
| $g_5$ | | For all $(x_n) \longrightarrow x_0$, $\lim_{n\to\infty} g_5(x_n) = -1$. | C |

6.

| $f_i$ | $f_i(x_0)$ | Behavior of $(f_i(x_n))_{n=1}^{\infty}$, where $(x_n)_{n=1}^{\infty}$ converges to $x_0$ | Continuous at $x_0$? |
|---|---|---|---|
| $f_1$ | | | |
| $f_2$ | | | |
| $f_3$ | | | |
| $f_4$ | | | |
| $f_5$ | | | |

**Your statement of the sequence-based definition of continuity:**